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# RADIATIVE HEAT TRANSFER NEAR THE STAGNATION POINT OF A BLUNT BODY

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# RADIATIVE HEAT TRANSFER NEAR THE STAGNATION POINT OF A BLUNT BODY

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Problems related to calculation of radiative heat transfer near the stagnation point of a two-dimensional or axisymmetric body in hypersonic flow are considered. They are, namely: 1) the effect of radiative heat transfer on convective heat transfer; 2) calculation of the radiant heat flux from a certain volume of gas toward a given point on the body. Under certain assumptions with respect to the radiation field, the problem is reduced to a form convenient for computer programming. A system of equations is obtained which determines the heat flux toward the wall due to the heat conductivity of the gas and to radiation by using the hypothesis of a two-dimensional parallel layer which in this case replaces the region between the shock wave and body and whose width is equal to the shock layer. Then, the problem of calculating the radiant heat flux toward a certain region near the stagnation point of a sphere is analyzed without using the hypothesis of a two-dimensional parallel layer. Formulas for computing the mutual effects of radiative heat transfer and heat conductivity, and for radiant heat flow to the neighborhood of the stagnation point are obtained.

* Numbers in the margin indicate pagination in the original foreign text.

This paper considers questions connected with the calculation of radiative heat transfer in the neighborhood of the stagnation point of a two-dimensional or axisymmetric body in hypersonic flow:

1) the effect of radiative heat transfer on convective heat transfer;
2) calculation of the radiant heat flux at a given point of the body about which a certain volume of gas circulates.

Under the usual assumptions relative to the radiation field (effects of scattering, radiation pressure, and density of radiant energy all assumed as small; gas in a state close to chemical and thermodynamic equilibrium), the problem reduces to a form convenient for computer programing.

The flow of the radiating gas between the shock wave and the body is described by the Navier-Stokes equations, with a radiation term in the equation of energy and in the equation of radiative transfer. Under the above assumptions, these equations may be written in the form of

$$\left.\begin{aligned} \frac{dJ_\nu^{(1)}}{ds} &= \frac{\partial J_\nu^{(1)}}{\partial x}\sin\theta + \frac{\partial J_\nu^{(1)}}{\partial y}\cos\theta = \alpha_\nu (B_\nu - J_\nu^{(1)}), \\ \frac{dJ_\nu^{(2)}}{dS} &= \frac{\partial J_\nu^{(2)}}{\partial x}\sin\theta + \frac{\partial J_\nu^{(2)}}{\partial y}\cos\theta = -\alpha_\nu (B_\nu - J_\nu^{(2)}), \end{aligned}\right|$$

where $J_\nu$ is the intensity of radiation, the superscript 1 relates to rays traveling from right to left (with the angle $\theta$ varying from 0 to $\frac{\pi}{2}$); the volume superscript 2 denotes rays traveling from left to right $\left(\theta \in \left[\frac{\pi}{2}, \pi\right]\right)$; $\alpha_\nu$ is the absorption coefficient; $\theta$ is the angle between the direction of the radiation and the ordinate axis; s represents the direction of radiation. The flux of radiant energy of the $\nu$-th frequency of the k-direction in projection onto the i axis will then be

$$H_{\nu i}^{(k)} = 2\pi \int_0^{\pi/2} J_\nu^{(k)} \cos(\widehat{s, i}) \sin\theta d\theta. \tag{1}$$

On the boundary of the region under investigation (in this case: body to shock wave), the conditions for the intensity of radiation $J_\nu^{(1,2)}$ must be prescribed. Thus, the calculation of the radiation in problems of gas dynamics makes it necessary to investigate the entire flow region in one unit. The solution of this general problem will answer all questions connected with radiative heat transfer.

In view of the complexity of the problem, various simplifying assumptions must be made. Earlier authors (Bibl.2 - 5) reported on studies of the flow /121 of the radiating gas in the neighborhood of the axial line. We will use the following scheme to describe the flow in the vicinity of the frontal point of a blunt body. We will consider that the shock wave is so thin that we can neglect the effects of curvature, pose $\frac{\partial P}{\partial y} = 0$, and determine the pressure variations in the direction of the x axis by the precise Newton formula

$$\frac{\partial P}{\partial x} = -\left(\frac{\sqrt{2bk}}{R} u_\infty\right)^2 x \rho_\Delta,$$

where b is an empirical constant equal to unity for a sphere (Bibl.5), $k = \frac{\rho_\infty}{\rho_\Delta}$. For the radiation, we will adopt the hypothesis of a plane-parallel layer often used in astrophysics. This stipulates that the radiation is from a plane layer with constant T and P, on lines parallel to the boundary. In our case, we will replace the entire region between the shock wave and the body by a plane layer of a thickness equal to the thickness of the shock layer at the stagnation point, and let the thermodynamic quantities vary only along the depth of the layer. Then, by virtue of the symmetry of the flow relative to the oy axis (the ox axis is directed along the body, the oy axis along the normal to the body at the frontal point) and confining the calculation to terms of the order of x, we will have

$$\rho=\rho(y),\ T=T(y),\ h=h(y),\ v=v(y),\ r\sim x,$$
$$\mu=\mu(y),\ J_\nu^{(k)}=J_\nu^{(k)}(y),\ u=xu_1(y),\ \lambda=\lambda(y). \tag{2}$$

We present below an expression for the radiant heat flux at the frontal point from a spherical segment with known distribution of the parameters. A comparison with the radiant flux from a plane layer may be of interest for evaluating the hypothesis of the plane-parallel layer.

1. Thus, under the assumptions adopted, the system of equations describing the flow in the neighborhood of the frontal point will have the form

$$\begin{gathered}\frac{\partial r^\varepsilon u\rho}{\partial x}+\frac{\partial r^\varepsilon v\rho}{\partial y}=0,\\ \rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right),\\ \rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=Au\frac{\partial P}{\partial x}-A\frac{\partial}{\partial y}\left(\frac{\mu}{2}\frac{\partial u^2}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}}\frac{\partial h}{\partial y}\right)-\frac{\partial H_y}{\partial y},\end{gathered} \tag{3}$$

$$\begin{gathered}H_y=2\pi\int_0^\infty\int_0^{\pi/2}\left(J_\nu^{(1)}-J_\nu^{(2)}\right)\sin\theta\cos\theta\, d\theta\, d\nu=H_y^{(1)}-H_y^{(2)},\\ \frac{dJ_\nu^{(1)}}{dy}\cos\theta=\alpha_\nu\left(B_\nu-J_\nu^{(1)}\right),\\ \frac{dJ_\nu^{(2)}}{dy}\cos\theta=-\alpha_\nu\left(B_\nu-J_\nu^{(2)}\right),\end{gathered} \tag{4}$$

with satisfaction of the conditions of dynamic compatibility on the shock-wave front and of the conditions of adhesion to the wall:

$$\begin{gathered}\text{at}\ \ y=\Delta,\ h=h_\Delta,\ u=\frac{u_\infty}{R+\Delta}x,\ v=\frac{-\rho_\infty u_\infty}{\rho_\Delta};\\ \text{at}\ \ y=0,\ u=v=0,\ h=h_w.\end{gathered} \tag{5}$$

Here $B_\nu$ denotes the Planck function; $\Delta$ is the thickness of the shock layer; R is the radius of the sphere; $\varepsilon$ is zero for two-dimensional bodies and unity for axisymmetric ones; the subscripts refer to the parameters: $\Delta$ beyond the shock wave, w on the wall, $\infty$ in the relative flow. The remainder of the notation /122 is standard. One of the conditions (5) is used to determine the unknown thickness of the shock layer $\Delta$.

Let us take the absorptance of the boundary of the shock wave as equal to unity, that of the wall as $\beta$, and consider the intensity $J_\nu^{(2)}(\Delta)$ to be known. Then, the condition of balance of the radiant energy on the wall will give the missing condition for the radiation intensity:

$$
\begin{aligned}
&\text{at } y=0,\ J_\nu^{(1)}(0)=\beta B_{\nu w}+(1-\beta)J_\nu^{(2)}(0);\\
&\text{at } y=\Delta,\ J_\nu^{(2)}(\Delta)=C_{\nu\Delta}.
\end{aligned}
\tag{5'}
$$

Passing to new variables, we have

$$
\begin{gathered}
u=\bar{x}f'(\eta)Rd,\ x=R\bar{x},\ J_\nu^{(1,2)}=\sigma T_\Delta^4\bar{J}_\nu^{(1,2)},\ C_{\nu\Delta}=\sigma T_\Delta^4\bar{C}_{\nu\Delta},\\
\rho=\rho_\Delta\bar{\rho},\ \mu=\mu_\Delta\bar{\mu},\ B_\nu=\sigma T_\Delta^4\bar{B}_\nu,\ \alpha_\nu=\alpha_{\nu\Delta}\bar{\alpha}_\nu,\ \eta=\int_0^{\bar{y}}\bar{\rho}\,d\bar{y},\\
y=\sqrt{\frac{\nu_\Delta}{d}}\,\bar{y},\ h=h_\Delta\bar{h},\ v=\sqrt{\nu_\Delta d}\,\bar{v},\ d=\frac{\sqrt{2bk}}{R}u_\infty.
\end{gathered}
$$

On the basis of eqs.(2), we can rewrite the system (3) - (4) with the boundary conditions (5) - (5') in the following form:

$$
\begin{gathered}
\bar{\rho}\bar{v}=-(\varepsilon+1)f(\eta),\\
(Kf'')'+(\varepsilon+1)ff''=f'^2-\frac{1}{\bar{\rho}},\\
\left(\frac{K}{\Pr}\bar{h}'\right)'+(\varepsilon+1)f\bar{h}'=B_0\frac{d}{d\eta}\left(\bar{H}_y^{(1)}-\bar{H}_y^{(2)}\right),
\end{gathered}
\tag{6}
$$

$$
\begin{gathered}
\frac{dJ_\nu^{(1)}}{d\eta}\cos\theta=a_{1\nu}\frac{\bar{\alpha}_\nu}{\bar{\rho}}\left(\bar{B}_\nu-\bar{J}_\nu^{(1)}\right),\\
\frac{dJ_\nu^{(2)}}{d\eta}\cos\theta=-a_{1\nu}\frac{\bar{\alpha}_\nu}{\bar{\rho}}\left(\bar{B}_\nu-\bar{J}_\nu^{(2)}\right),
\end{gathered}
\tag{7}
$$

where

$$
B_0=\frac{\sigma T_\Delta^4}{\rho_\Delta h_\Delta\sqrt{\nu_\Delta d}},\quad a_{1\nu}=\frac{\alpha_{\nu\Delta}\sqrt{\nu_\Delta}}{\sqrt{d}},\quad K=\bar{\rho}\bar{\mu}.
$$

$$
\begin{gathered}
\text{At}\quad \eta=\eta_\Delta,\ f'=\frac{u_\infty}{(R+\Delta)d},\ f=\frac{ku_\infty}{(1+\varepsilon)\sqrt{\nu_\Delta d}},\ \bar{h}=1,\\
\text{at}\quad \eta=0,\ f=f'=0,\ \bar{h}=\bar{h}_w,
\end{gathered}
\tag{8}
$$

$$
\begin{gathered}
\bar{J}_\nu^{(1)}(0)=\beta\bar{B}_{\nu w}+(1-\beta)\bar{J}_\nu^{(2)}(0),\\
\bar{J}_\nu^{(2)}(\eta_\Delta)=\bar{c}_{\nu\Delta}.
\end{gathered}
\tag{9}
$$

The formal solution of eqs.(7) with the boundary conditions (9) can be written in quadratures.

Elementary transformations will yield the following expression for $\frac{d\bar{H}_y}{d\eta}$:

$$\frac{d\overline{H}_y}{d\eta}=2\pi a_{1\,\max}\int_0^{\infty}\overline{a}_{1\nu}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}\Bigg\{2\overline{B}_\nu-a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\overline{\rho}}}E^{(-1)}\left(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta-a_{1\nu}\times\right.$$

$$\left.\times\int_0^{\eta_1}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta_1\right)d\eta_1-a_{1\nu}\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\overline{\rho}}}E^{(-1)}\left(a_{1\nu}\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta-a_{1\nu}\int_{\eta_1}^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta_1\right)d\eta_1-$$

$$-a_{1\nu}(1-\beta)\int_0^{\eta_\Delta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\overline{\rho}}}E^{(-1)}\left(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta+a_{1\nu}\int_0^{\eta_1}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta_1\right)d\eta_1-$$ /123

$$-\overline{C}_{\nu\Delta}E^{(-2)}\left(a_{1\nu}\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta\right)-(1-\beta)\,\overline{C}_{\nu\Delta}E^{(-2)}\left(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta+\right.$$

$$\left.+a_{1\nu}\int_0^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta\right)-\beta\overline{B}_{\nu w}E^{(-2)}\left(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}d\eta\right)\Bigg\}d\nu, \tag{10}$$

where

$$\overline{a}_{1\nu}=\frac{a_{1\nu}}{a_{1\,\max}},\quad a_{1\,\max}=\frac{a_{\max\Delta}\sqrt{\nu_\Delta}}{\sqrt{d}},\quad E^{(\pm n)}(b)=\int_1^{\infty}t^{\pm n}e^{-bt}dt.$$

The method of solution of the system (6) with the boundary conditions (8) is as follows: Let us represent the functions, depending on $\overline{h}$ and entering into the system (6), in the form of polynomials of a certain degree

$$K=\sum_{j=0}^{k_1}d_j\overline{h}^j,\quad \frac{1}{\overline{\overline{\rho}}}=\sum_{j=0}^{k_2}b_j\overline{h}^j,\quad \frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\overline{\rho}}}=\sum_{j=0}^{k_3}c_{\nu j}\overline{h}^j,\quad \frac{\overline{\alpha}_\nu}{\overline{\overline{\rho}}}=\sum_{j=0}^{k_4}d_{\nu j}\overline{h}^j. \tag{11}$$

Let us then expand the required functions f and $\overline{h}$ in series in the parameter $\overline{B}_0$

$$f=\sum_{n=0}^{\infty}\overline{B}_0^nf_n,\quad \overline{h}=\sum_{n=0}^{\infty}\overline{B}_0^n\overline{h}_n,\quad \overline{B}_0=a_{1\,\max}B_0. \tag{12}$$

Substituting the series (12) into the system (6), we obtain recurrent equations for the determination of $f_\ell$ and $\overline{h}_\ell$, for arbitrary $\ell$.

Hereafter, we will use the following identities, which can be proved by elementary transformations using the method of mathematical induction (for simplicity, we will omit the vinculum over the required functions):

$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_j=0}^{\infty}\bar{B}_0^{n_1+n_2+\dots+n_j}h_{n_1}h_{n_2}\dots h_{n_j}=$$
$$=\sum_{l=0}^{\infty}\bar{B}_0^{l}\sum_{n_1=0}^{l}\sum_{n_2=0}^{l-n_1}\cdots\sum_{n_{j-1}=0}^{l-n_1-\dots-n_{j-2}}h_{n_1}h_{n_2}\dots h_{n_{j-1}}h_{l-n_1-\dots-n_{j-1}}, \qquad (13.1)$$

$$\sum_{n_1=0}^{1}\sum_{n_2=0}^{1-n_1}\cdots\sum_{n_{j-1}=0}^{1-n_1-\dots-n_{j-2}}h_{n_1}h_{n_2}\dots h_{n_{j-1}}h_{1-n_1-\dots-n_{j-1}}=jh_1h_0^{j-1},$$
$$\sum_{n_1=0}^{2}\sum_{n_2=0}^{2-n_1}\cdots\sum_{n_{j-1}=0}^{2-n_1-\dots-n_{j-2}}h_{n_1}h_{n_2}\dots h_{n_{j-1}}h_{2-n_1-\dots-n_{j-1}}=jh_2h_0^{j-1}+$$
$$+\frac{1}{2}j(j-1)h_1^2h_0^{j-2}. \qquad (13.2)$$

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$$\sum_{n_1=0}^{l}\sum_{n_2=0}^{l-n_1}\cdots\sum_{n_{j-1}=0}^{l-n_1-\dots-n_{j-2}}h_{n_1}h_{n_2}\dots h_{n_{j-1}}h_{l-n_1-\dots-n_{j-1}}=$$
$$=\sum_{m=1}^{l}\frac{j(j-1)\dots(j-m+1)}{m!}h_0^{j-m}\sum_{n_1=1}^{s_1^{(l,m)}}\sum_{n_2=1}^{s_2^{(l,m)}}\cdots\sum_{n_{m-1}=1}^{s_{m-1}^{(l,m)}}h_{n_1}\times$$
$$\times\dots h_{n_{m-1}}h_{l-n_1-\dots-n_{m-1}},$$

where

$$s_1^{(l,m)}=l-m+1,\ s_2^{(l,m)}=l-m-n_1+2,\ \dots,\ s_{m-1}^{(l,m)}=$$
$$=l-m-n_1-\dots-n_{m-2}+m-1.$$

The equations for the determination of $f_l$ and $h_l$ will have the form

$$[K(h_0)f_0'']'+(\varepsilon+1)f_0f_0''=f_0'^2-\frac{1}{\rho(h_0)}, \qquad (14.0)$$
$$\left[\frac{K}{\Pr}(h_0)h_0'\right]'+(\varepsilon+1)f_0h_0'=0,$$
$$\eta=0,\ f=f'=0,\ h=h_w;$$
$$\eta=\eta_\infty,\ f'=\frac{u_\infty k}{(1+\varepsilon)\sqrt{\nu_\infty d}},\ h=1.$$

$$f_1'''K(h_0)+(1+\varepsilon)f_0f_1''+f_1''h_0'\frac{dK}{dh}(h_0)-2f_0'f_1'+(1+\varepsilon)f_1f_0''+$$
$$+f_0''h_1'\frac{dK}{dh}(h_0)+h_1\left[f_0'''\frac{dK}{dh}(h_0)+f_0''h_0'\frac{d^2K}{dh^2}(h_0)+\frac{d}{dh}\frac{1}{\rho}(h_0)\right]=0, \qquad (14.1)$$
$$h_1''\frac{K}{\Pr}(h_0)+\left[2h_0'\frac{d}{dh}\frac{K}{\Pr}(h_0)+(1+\varepsilon)f_0\right]h_1'+$$
$$+h_1\left[h_0''\frac{d}{dh}\frac{K}{\Pr}(h_0)+h_0'^2\frac{d^2}{dh^2}\frac{K}{\Pr}(h_0)\right]+(1+\varepsilon)h_0'f_1=L^{(1)}(\eta),$$
$$\eta=0,\ f=f'=0,\ h=0;$$

$$\eta=\eta_\delta,\ f=0,\ h=0.$$

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$$\sum_{n_1=0}^{l}\sum_{m=1}^{n_1}\frac{1}{m!}\frac{d^m}{dh^m}K(h_0)P_{m-1}^{(n_1)}f'''_{l-n_1}+\sum_{n_1=0}^{l}\sum_{n_2=0}^{l-n_1}\sum_{m=1}^{n_1}\frac{1}{m!}\cdot$$

$$\cdot\frac{d^{m+1}}{dh^{m+1}}K(h_0)P_{m-1}^{(n_1)}f''_{n_2}h'_{l-n_1-n_2}+(1+\varepsilon)\sum_{n=0}^{l}f_nf''_{l-n}=$$

$$=\sum_{n=0}^{l}f'_nf'_{l-n}-\sum_{m=1}^{l}\frac{1}{m!}\frac{d^m}{dh^m}\frac{1}{\rho}(h_0)P \ldots \tag{14.$\ell$}$$

$$\sum_{n_1=0}^{l}\sum_{m=1}^{n_1}\frac{1}{m!}\frac{d^m}{dh^m}\frac{K}{\Pr}(h_0)P_{m-1}^{(n_1)}h''_{l-n_1}+\sum_{n_1=0}^{l}\sum_{n_2=0}^{l-n_1}\sum_{m=1}^{n_1}\frac{1}{m!}\frac{d^{m+1}}{dh^{m+1}}\frac{K}{\Pr}(h_0)\times$$

$$\times P_{m-1}^{(n_1)}h'_{n_2}h'_{l-n_1-n_2}+(1+\varepsilon)\sum_{n_1=0}^{l}f_{n_1}h'_{l-n_1}=L^{(l)}(\eta),$$

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$$l\geqslant 1,$$
$$\eta=0,\ f=f'=0,\ h=0;$$
$$\eta=\eta_\delta,\ f=0,\ h=0,$$

where

$$P_{m-1}^{(l)}=\sum_{n_1=1}^{s_1^{(l,m)}}\sum_{n_2=1}^{s_2^{(l,m)}}\cdots\sum_{n_{m-1}=1}^{s_{m-1}^{(l,m)}}h_{n_1}\ldots h_{n_{m-1}}h_{l-n_1-\ldots-n_{m-1}},$$

Here, $L^{(\ell)}(\eta)$ is the coefficient of the term of degree $\bar{B}_0^{\ell}$ in the expansion in $\bar{B}_0$ of the right-hand side of the equation of energy [eq.(6)].

It is customary to use the following symbolic notation in writing eqs.(14):

$$\sum_{m=1}^{0}\frac{1}{m!}\frac{d^m}{dh^m}a(h_0)P_{m-1}^{(0)}=a(h_0),$$

$$\sum_{m=1}^{0}\frac{2}{m!}\frac{d^{m+1}}{dh^{m+1}}a(h_0)P_{m-1}^{(0)}=\frac{d}{dh}a(h_0).$$

Here and in what follows, the expressions $a(h_0)$ and $\frac{d^m}{dh^m}a(h_0)$ mean that the known functions $a(h)$ and $\frac{d^m}{dh^m}a(h) = F(h)$ are found as functions of $\eta$ from the previously determined functions $h_0 = h_0(\eta)$. Let us determine $L^{(0)}(\eta)$, $L^{(1)}(\eta)$, ..., $L^{(\ell)}(\eta)$, ... .

As an example let us expand in a series in $\overline{B}_0$ the function $e^{-\frac{a_{1\nu}}{\cos\theta}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}d\eta}$.
Elementary transformations, using the identities (13.1) - (13.2), will then yield

$$e^{-\frac{a_{\nu 1}}{\cos\theta}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}d\eta} = \sum_{n=0}^{\infty}\frac{(-1)^n a_{1\nu}^n}{n!\cos^n\theta}\left(\sum_{m=0}^{\infty}\overline{B}_0^m A_{k_1}^{(m)}\right)^n = \sum_{l=0}^{\infty}\overline{B}_0^l\sum_{m=1}^{l}\frac{1}{m!}\times$$

$$\times\frac{d^m}{dZ^m}e^{-\frac{a_{1\nu}}{\cos\theta}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta}\sum_{n_1=1}^{s_1^{(l,m)}}\sum_{n_2=1}^{s_2^{(l,m)}}\cdots\sum_{n_{m-1}=1}^{s_{m-1}^{(l,m)}}A_{k_1}^{(n_1)}\ldots A_{k_1}^{(n_{m-1})}A_{k_1}^{(l-n_1-\ldots-n_{m-1})},$$

where

$$Z=\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta,\quad A_{k_1}^{(m)}=\sum_{j=0}^{k_1}\sum_{n_1=0}^{m}\sum_{n_2=0}^{m-n_1}\cdots\sum_{n_{j-1}=0}^{m-n_1-\ldots-n_{j-2}}\int_0^{\eta}d_{\nu j}h_{n_1}\ldots$$

$$\ldots h_{n_{j-1}}h_{m-n_1-\ldots-n_{j-1}}d\eta,$$

for which the following symbolic notation is customarily used:

$$\sum_{n_1=0}^{l}\sum_{n_2=0}^{l-n_1}\cdots\sum_{n_{j-1}=0}^{l-n_1-\ldots-n_{j-2}}C_{n_1}\ldots C_{n_{j-1}}C_{l-n_1-\ldots-n_{j-1}}=$$

$$=\begin{cases}0 & \text{at } j=0,\ l>0,\\ C_0 & \text{at } j=0,\ l=0.\end{cases}$$

Performing the corresponding operations with all terms in eq.(10) and making /126 use of the connectivity between the $\ell$-th term of the expansion in series in $\overline{B}_0$ of the product $(A\cdot C)_{\overline{B}_0^{\ell}}$ as well as of the known terms of the expansion of its cofactors $(A)_{\overline{B}_0^m}$ and $(C)_{\overline{B}_0^m}$ $(m = 0, 1, \ldots, \ell)$

$$(AC)_{\overline{B}_0^l}=\sum_{n=0}^{l}(A)_{\overline{B}_0^n}(C)_{\overline{B}_0^{l-n}},$$

we obtain the following expressions for $L^{(0)}(\eta)$, $L^{(1)}(\eta)$, ..., $L^{(\ell)}(\eta)$, ...:

$$L^{(0)}(\eta)=0,$$

$$L^{(1)}(\eta)=2\pi\int_0^{\infty}\overline{a}_{1\nu}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\Big\{2\overline{B}_\nu-a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\rho}}(h_0)\,E^{(-1)}\Big(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}}{\overline{\rho}}(h_0)\,d\eta-$$

$$-a_{1\nu}\int_0^{\eta_1}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta_1\Big)-a_{1\nu}\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\rho}}(h_0)\,E^{(-1)}\Big(a_{1\nu}\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta-a_{1\nu}\times$$

$$\times\int_{\eta}^{\eta_\Delta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta_1\Big)d\eta_1-a_{1\nu}(1-\beta)\int_0^{\eta_\Delta}\frac{\overline{\alpha}_\nu\overline{B}_\nu}{\overline{\rho}}(h_0)\,E^{(-1)}\Big(a_{1\nu}\int_0^{\eta}\frac{\overline{\alpha}_\nu}{\overline{\rho}}(h_0)\,d\eta+$$

$$+\bar{a}_{1\nu}\int_0^{\eta_1}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)d\eta_1-\bar{C}_{\nu\Delta}E^{(-2)}\Bigg(a_{1\nu}\int_\eta^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)-(1-\beta)\,\bar{C}_{\nu\Delta}\times$$

$$\times E^{(-2)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta+a_{1\nu}\int_0^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)-\beta B_{\nu w}E^{(-2)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)\Bigg\}\,d\nu,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (15)$$

$$L^{(l+1)}(\eta)=4\pi\int_0^\infty\bar{a}_{1\nu}\sum_{m=1}^{l}\frac{d^m}{dh^m}\frac{\bar{\alpha}_\nu\bar{B}_\nu}{\bar{\rho}}(h_0)\,P^{(l)}_{m-1}-2\pi\int_0^\infty a_{1\nu}\sum_{n=0}^{l}\sum_{m_1=1}^{n}\frac{1}{m_1!}\times$$

$$\times\frac{d^{m_1}}{dh^{m_1}}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,P^{(n)}_{m_1-1}\Bigg\{\int_0^\eta\sum_{s=0}^{l-n}\Bigg[\sum_{m_2=1}^{s}\frac{1}{m_2!}\frac{d^{m_2}}{dh^{m_2}}\frac{\bar{\alpha}_\nu\bar{B}_\nu}{\bar{\rho}}(h_0)\,P^{(s)}_{m_2-1}\Bigg]\Bigg[\sum_{m_3=1}^{l-n-s}\frac{(-1)^{m_3}a_{1\nu}^{m_3}}{m_3!}\times$$

$$\times E^{(m_3-1)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta-a_{1\nu}\int_0^{\eta_1}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta_1\Bigg)D^{(l-n-s)}_{m_3}(\eta-\eta_1)\Bigg]d\eta_1+$$

$$+\int_\eta^{\eta_\Delta}\sum_{s=0}^{l-n}\Bigg[\sum_{m_2=1}^{s}\frac{1}{m_2!}\frac{d^{m_2}}{dh^{m_2}}\frac{\bar{\alpha}_\nu\bar{B}_\nu}{\bar{\rho}}(h_0)\,P^{(s)}_{m_2-1}\Bigg]\Bigg[\sum_{m_3=1}^{l-n-s}\frac{(-1)^{m_3}a_{1\nu}^{m_3}}{m_3!}\times$$

$$\times E^{(m_3-1)}\Bigg(a_{1\nu}\int_\eta^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta-a_{1\nu}\int_{\eta_1}^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta_1\Bigg)D^{(l-n-s)}_{m_3}(\eta_1-\eta)\Bigg]d\eta_1+$$

$$+(1-\beta)\int_0^{\eta_\Delta}\sum_{s=0}^{l-n}\Bigg[\sum_{m_2=1}^{s}\frac{1}{m_2!}\frac{d^{m_2}}{dh^{m_2}}\frac{\bar{\alpha}_\nu\bar{B}_\nu}{\bar{\rho}}(h_0)\,P^{(s)}_{m_2-1}\Bigg]\Bigg[\sum_{m_3=1}^{l-n-s}\frac{(-1)^{m_3}a_{1\nu}^{m_3}}{m_3!}\times$$

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$$\times E^{(m_3-1)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta+a_{1\nu}\int_0^{\eta_1}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta_1\Bigg)D^{(l-n-s)}_{m_3}(\eta+\eta_1)\Bigg]d\eta_1\Bigg\}-$$

$$-2\pi\int_0^\infty\bar{a}_{1\nu}\sum_{n=0}^{l}\sum_{m_1=1}^{n}\frac{1}{m_1!}\frac{d^{m_1}}{dh^{m_1}}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,P^{(n)}_{m_1-1}\Bigg\{\bar{C}_{\nu\Delta}\sum_{m_2=1}^{l-n}\frac{(-1)^{m_2}a_{1\nu}^{m_2}}{m_2!}\times$$

$$\times E^{(m_2-2)}\Bigg(a_{1\nu}\int_\eta^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)D^{(l-n)}_{m_2}(\eta_\Delta-\eta)+(1-\beta)\,\bar{C}_{\nu\Delta}\sum_{m_2=1}^{l-n}\frac{(-1)^{m_2}a_{1\nu}^{m_2}}{m_2!}\times$$

$$\times E^{(m_2-2)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta+a_{1\nu}\int_0^{\eta_\Delta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)D^{(l-n)}_{m_2}(\eta+\eta_\Delta)+$$

$$+\beta B_{\nu w}\sum_{m_2=1}^{l-n}\frac{(-1)^{m_2}a_{1\nu}^{m_2}}{m_2!}E^{(m_2-2)}\Bigg(a_{1\nu}\int_0^{\eta}\frac{\bar{\alpha}_\nu}{\bar{\rho}}(h_0)\,d\eta\Bigg)D^{(l-n)}_{m_2}(\eta)\Bigg\}\,d\nu,$$

where

$$D^{(l)}_m(x\pm y)=\sum_{n_1=1}^{s_1^{(l,m)}}\sum_{n_2=1}^{s_2^{(l,m)}}\cdots\sum_{n_{m-1}=1}^{s_{m-1}^{(l,m)}}A^{(n_1)}_{k_1}(x\pm y)\ldots A^{(n_{m-1})}_{k_1}(x\pm y)\times$$

$$\times A_{h_{n_1}}^{(l-n_1-\ldots-n_{m-1})}(x \pm y);$$

$$A_{h_1}^{(m)}(x \pm y) = \sum_{j=0}^{h_1} d_{\nu j} \sum_{n_1=0}^{m} \sum_{n_2=0}^{m-n_1} \ldots \sum_{n_{j-1}=0}^{m-n_1-\ldots-n_{j-2}} \times$$

$$\times \left[ \int_0^x h_{n_1} \ldots h_{n_{j-1}} h_{m-n_1-\ldots-n_{j-1}} d\eta \pm \int_0^y h_{n_1} \ldots h_{n_{j-1}} h_{m-n_1-\ldots-n_{j-1}} d\eta \right].$$

The expressions for $L^{(2)}(\eta)$ and $L^{(3)}(\eta)$ can be written in more compact form:

$$L^{(2)}(\eta) = [L^{(1)}(\eta)]', \quad L^{(3)}(\eta) = \left[\frac{1}{2} L^{(2)}(\eta)\right]',$$

where the sign [ ]' denotes the symbol derivative

$$[C]' = h_1 \frac{dC}{dh}(h_0), \quad \left[h_1 \frac{dC}{dh}\right]' = 2h_2 \frac{dC}{dh}(h_0) + h_1^2 \frac{d^2C}{dh^2}(h_0).$$

Thus, the problem of the flow of a radiating gas in the neighborhood of the axial line has been reduced to the solution of eqs.(14) which, for $l$ = 1, 2, ..., are a system of inhomogeneous ordinary linear equations of the type

$$a_1(\eta) f_l''' + a_2(\eta) f_l'' + a_3(\eta) f_l' + a_4(\eta) f_l + a_5(\eta) h_l' + a_6(\eta) h_l = a_7(\eta),$$
$$b_1(\eta) h_l'' + b_2(\eta) h_l' + b_3(\eta) h_l + b_4(\eta) f_l = b_5(\eta),$$

where all the coefficients $a(\eta)$ and $b(\eta)$ are known functions of $\eta$. The solution of eqs.(14) will determine the heat flux to the wall, due both to thermal conduction by the gas and to radiation. The latter is represented by eq.(1) for k = 2 as

$$\overline{H}_y^{(2)}(0) = 2\pi \int_0^\infty \int_0^{\pi/2} \overline{J}_\nu^{(2)}(0) \sin\theta \cos\theta \, d\theta d\nu =$$ /128

$$= 2\pi \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\frac{a_{1\nu}}{\cos\theta} \int_0^{\eta_\Delta} \frac{\bar{\alpha}_\nu}{\rho} d\eta} \left[ a_{1\nu} \int_0^{\eta_\Delta} \frac{\bar{\alpha}_\nu \bar{B}_\nu}{\rho} e^{\frac{a_{1\nu}}{\cos\theta} \int_\eta^{\eta_\Delta} \frac{\bar{\alpha}_\nu}{\rho} d\eta} + \overline{C}_{\nu\Delta} \right] \sin\theta \, d\theta d\nu. \quad (16)$$

If we assume that the radiation has no effect on the velocity profile nor on the departure of the shock wave, and if we set K = const, Pr = const, the problem will be greatly simplified; in this case, we need only solve the system (14.0) and perform quadratures:

$$h_1 = C_1 \int_0^{\eta} e^{-\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} d\eta + \frac{\Pr}{K}\int_0^{\eta} e^{-\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} \times$$

$$\times \int_0^{\eta} L^{(1)}(\eta)\, e^{\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} d\eta d\eta,$$

where

$$C_1 = \frac{-\frac{\Pr}{K}\int_0^{\eta_\Delta} e^{-\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} \int_0^{\eta} L^{(1)}(\eta)\, e^{\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} d\eta d\eta}{\int_0^{\eta_\Delta} e^{-\frac{(1+\epsilon)\Pr}{K}\int_0^{\eta} f d\eta} d\eta}. \quad (17)$$

2. We now consider the question of calculating the radiant heat flux in a certain neighborhood of the leading edge of a sphere circumflowed by a hypersonic flow, without using the plane-parallel layer hypothesis for the radiation.

Let us assume that the shape of the shock wave is prescribed (to simplify the reasoning, let us consider that there is a sphere of radius $R + \Delta$ up to a certain point $A(x_0, y_0, z_0)$, $z_0 \leq R$), and that we know the gas-dynamic and thermodynamic parameters between the shock wave and the body. Because of the symmetry of the radiating volume, the radiant heat flux will be the same along circles with their center at the critical point and lying on the sphere 1. We can therefore consider the plane projection of our volume in the first quadrant of the vertical plane xoz (see diagram). Let us restrict the calculation to considering only those points on the sphere in this gas flow, where the tangent planes to these points cut off spherical segments from the radiating volume. Obviously, if we produce the tangent from the point A to the circle of a radius R, then the coordinates of the point of tangency $B(X, Z)$ will determine the boundary of the region to be investigated on the sphere. Elementary transformations will yield

$$X_{1,2} = \frac{-k_{1,2} b_{1,2}}{1+k_{1,2}^2}, \quad Z_{1,2} = k_{1,2} X_{1,2} + b_{1,2},$$

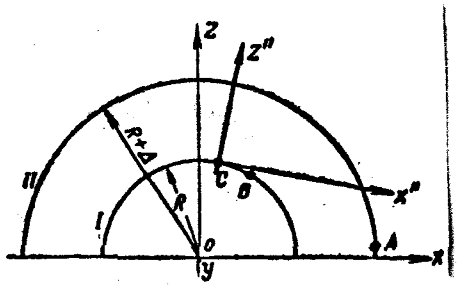
where

$$k_{1,2}=\frac{-z_0x_0\pm R\sqrt{x_0^2+z_0^2-R^2}}{R^2-x_0^2},\qquad b_{1,2}=\mp\frac{Rx_0\sqrt{x_0^2+z_0^2-R^2}}{R^2-x_0^2},$$

and we must use here that sign in the formulas for $k_{1,2}$ and $b_{1,2}$ which will make $X_{1,2} > 0$ and $Z_{1,2} > 0$. The expression for the radiant heat flux from an arbitrary volume to an arbitrary point has been given previously (Bibl.1). For a "nongray" medium it has the form /129

$$q=\int_0^\infty\iiint_V a_\nu B_\nu \exp\left(-\int_0^r a_\nu dr\right)\sin\theta\cos\theta d\theta d\varphi dr d\nu. \qquad (18)$$

Let $x_1$ and $z_1$ be the coordinates of an arbitrary point on the circle of radius R, belonging to the region [0, X; R, Z]. Let us transfer the origin of coordinates




to the point $C(x_1, z_1)$ and superpose the axis ox with the tangent to the circle I at the point C. The formulas of transition from the system of coordinates xyz to x"y"z" will then read

$$x=x_1+x''\cos\varphi_1-z''\sin\varphi_1,$$
$$z=z_1+z''\cos\varphi_1+x''\sin\varphi_1,$$
$$y=y'',$$

where $\varphi_1$ is the angle of slope of the tangent from the point C to the axis ox

$$\varphi_1=\tan^{-1}\left(-\frac{x_1}{z_1}\right).$$

We can now pass to a determination of the limits of integration over r, θ, and φ in eq.(18). In view of the asymmetry of the radiating volume relative to the plane x"Cz", it is obvious that the value of the angle φ lies in the region

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The shock wave, as far as the point A in the system $x''y''z''$, is determined by the equation

$$(x''-a)^2+y''^2+(z''-c)^2=(R+\Delta)^2,$$

where

$$a=-x_1\cos\varphi_1-z_1\sin\varphi_1,\quad c=x_1\sin\varphi_1-z_1\cos\varphi_1.$$

On variation of $\varphi$ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the wave forms, in the planes $x''= y''\tan\varphi$, segments whose arcs are determined by the equations

$$\left(\frac{x''}{\sin\varphi}-a\sin\varphi\right)^2+(z''-c)^2=(R+\Delta)^2-a^2\cos^2\varphi. \qquad (19)$$

Hence, it is obvious that $\theta\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, but that $r\in[0, r_1]$, where $r_1$ is the distance from the point $C(x_1, z_1)$ to the point of intersection of the straight line $z''=\frac{x''}{\sin\varphi}\cot\theta$ with the circle of eq.(19). The expression for $r_1$ then becomes

$$r_{1,2}=\left|a\sin\varphi\sin\theta+c\cos\theta\pm\right.$$
$$\left.\pm\sqrt{(a\sin\varphi\sin\theta+c\cos\theta)^2+(R+\Delta)^2-a^2-c^2}\right|.$$

The sign must be taken such that

$$z_{1,2}=\cos\theta\left[a\sin\varphi\sin\theta+c\cos\theta\pm\right.$$
$$\left.\pm\sqrt{(a\sin\varphi\sin\theta+c\cos\theta)^2+(R+\Delta)^2-a^2-c^2}\right]\geqslant 0.$$

Making use of the symbolic notation given earlier (Bibl.1), we can write /130 the expression for the radiant heat flux at the point C in the form of

$$q=\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\left[-\frac{\pi}{2}, \frac{\pi}{2}\right][0, r_1]. \qquad (20)$$

For the frontal point, we have $\varphi_1 = 0$, $x_1 = 0$, $z_1 = R$

$$r_{1,2}=\left|-R\cos\theta\pm\sqrt{R^2\cos^2\theta+(R+\Delta)^2-R^2}\right|,$$

where the sign must be so selected that

$$z_{1,2}=-R\cos^2\theta\pm\cos\theta\sqrt{R^2\cos^2\theta+(R+\Delta)^2-R^2}\geqslant 0$$

and eq.(20) can be written as

$$q = 2\pi \left[0, \frac{\pi}{2}\right] [0, r_1]. \tag{21}$$

Equation (21), together with eq.(16) permit an evaluation of the hypothesis of the plane-parallel layer and a more accurate determination of the radiant heat flux.

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